Lab 1: Experimental Uncertainty

Objectives:
- to become acquainted with the fundamental methods of error analysis
- to understand and be able to calculate the mean and standard deviation of a set of measurements
- to be able to combine uncertainties
- to be able to calculate fractional and percent uncertainty
- to understand accuracy and precision
- to learn how to use a micrometer and a Vernier caliper

Equipment;

cylinders of different densities
string
ruler
micrometer
Vernier caliper
beam balance

Exploration 1. Measurement

This is just an exercise to do while you are waiting for everyone to do Exploration 1.1. You do not need to do this before Exploration 1.1 and you should not use the Vernier calipers in Exploration 1.1. However, while you are waiting for each student in the class to carry out the measurements in Exploration 1.1, if you have not already, read Appendix I and learn how to use the Vernier caliper and micrometer. Take some measurements with the Vernier caliper and micrometer to make sure you know how to correctly read the instruments. Make sure all of your group members know how to read the instruments and ask your TA to check and see if you are properly reading the instruments.

Exploration 1.1 Each person in the class should independently determine the circumference, C, and diameter, D, of a specified cylinder using a low-precision instrument, such as a meter stick or ruler. You may also wish to use a piece of string or some other readily available equipment. DO NOT use the Vernier caliper for this part. (Everyone in the class will measure the same cylinder.) Each person's measurement of the circumference and diameter of the cylinder will be recorded in a table on the board. Your group should enter the results on the board into a spreadsheet in an Excel file. Record your individual results in the space below.

Exploration 1.2 Once you have recorded all of the measurements in a spreadsheet, determine how to calculate and report your best estimate for the circumference and your best estimate for the diameter. Describe your method and report your results in the space below.

Exploration 1.3 Is there any uncertainty in your best estimates for the circumference and diameter in Exploration 1.2? (Do you know the circumference and diameter exactly or could you give a reasonable range into which you think a measurement would fall?) Estimate the uncertainty in your estimates. Do this by determining a reasonable range into which you would expect a measurement to fall. This range is a measure of the uncertainty of your measurement. Describe the method you use to estimate the uncertainty. This does not have to be a formal uncertainty calculation. It can be a reasonable way to estimate the range into which a measurement would fall.

Exploration 1.4 Now determine your best estimate for the quantity C/D and the uncertainty in that estimate. How would you determine the uncertainty in this case? Describe your method for determining the uncertainty in the value C/D.

Exploration 1.5 The most common way to determine a best estimate from a number of measurements is to calculate the mean of the measurements. Mathematically, for a series of measurements x_i, the mean is calculated by summing the measurements and dividing by the total number of measurements:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Use Excel to calculate the mean of your diameter and circumference measurements, if you have not already done so.

A normal or Gaussian distribution is a distribution that looks like a Bell curve, symmetric and centered at the origin or the mean:
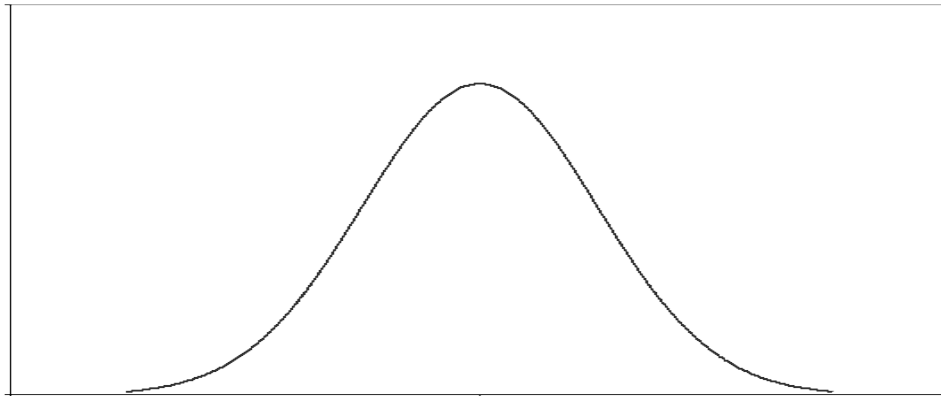
Mathematically, the *probability density function* for the standard Gaussian distribution with mean 0 and standard deviation 1 and the Gaussian distribution with mean μ and standard deviation σ are given by the following formulas:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

$$\phi(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

For a normal or Gaussian distribution, the most common method for determining the uncertainty is to use the standard deviation.

The deviation (d_i) of each value of the variable x_i from the mean is:

$$d_i = x_i - \bar{x}$$

The variance, σ^2, is the average value of the squared deviation of a set of N data points:

$$\sigma^2 = \frac{\sum_{i=1}^{N} d_i^2}{N-1}$$

The standard deviation, σ, is the square root of the variance:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} d_i^2}{N-1}}$$

Two thirds of the data fall within one standard deviation of the mean. For a large number of measurements, one can use

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} d_i^2}{N}}$$

to calculate the standard deviation because N >> 1.

Exploration 1.6 The most common best estimate for the uncertainty of a normal (Gaussian) distribution is the standard deviation. Use Excel to calculate the best estimate of the uncertainty of your diameter and circumference measurements. The best estimate is usually reported as the mean +/- the standard deviation. Record your results for the best estimate of the diameter and circumference in the form $\bar{x} +/- \sigma_x$.

Exploration 1.7 Use the instructions for the Propagation of Uncertainty in the Introduction to the lab manual and Excel to calculate the uncertainty in the quantity C/D. Show your work in the space below and record your best estimate for the value of C/D +/- (uncertainty in C/D).

Exploration 1.8 Do you have any expectations for the value of the quantity C/D? Is there an accepted value for the quantity C/D? Explain. Have you made a precise and accurate measurement of the quantity C/D? Explain.

Exploration 2 Graphical Methods

Exploration 2.1 Express your answer as your best estimate for C/D +/- the uncertainty in your best estimate. Can you express this graphically, indicating your best estimate and a possible range of this best estimate in a picture? Express your best estimate +/- the uncertainty graphically. How could your picture be used to determine how your result compares to the accepted value or to another person's best estimate +/- the uncertainty. Explain.

Exploration 2.2 Create a frequency distribution graph of each person's estimate of C/D on a piece of graph paper. (Choose a reasonable bin size. Divide the data into discrete bins and plot the number of data points in each bin.) Is the graph approximately Gaussian?

Exploration 2.3 Could the graph in Exploration 2.2 be used to express C/D as a number +/- the uncertainty in that number? How? Explain.

Exploration 3 Meaning of Uncertainty

Exploration 3.1 Calculate the fractional and percent uncertainty for your value of C/D, as in section 3.3 in the Introduction of the lab manual. Show your work and record your results below.

Exploration 3.2 Accuracy

Refer to the definition of accuracy in the Introduction to the lab manual. How accurate is your result? Explain.

Exploration 3.3 Precision

Refer to the definition of precision in the Introduction to the lab manual. How precise is your result? Explain.

Investigation 1 Density measurement

In this lab, you are going to make a precise measurement of the density of a cylinder at your table. You are going to measure the diameter, D, length, L, and mass, M, of the cylinder multiple times and determine the mean and standard deviation of each of the measurements. The means will be indicated as, $\overline{D}$, $\overline{L}$ and $\overline{M}$ and the standard deviations as, σ_D, σ_L, and σ_M.

Investigation 1.1 Diameter, length and mass

Investigation 1.1.a Use of Vernier calipers and micrometer

Read Appendix I on how to use the Vernier caliper and micrometer. Each person must explain how to use and demonstrate taking a reading to the TA before continuing. You must have the TA initial your lab at this point to verify that you know how to use the instruments, in order to continue.

Investigation 1.1.b Mean and standard deviation measurements

As a group, use the Vernier calipers, micrometer or balance beam to measure the diameter, length and mass of a cylinder at least ten times, with each person in your group contributing at least two measurements to each of the measurements of D, L and M. Record the data in the tables at the end of this lab or in an Excel spreadsheet. Use Excel to determine the mean and standard deviation of each of the measurements. Record the means and standard deviations in the tables at the end of the lab and record the means and uncertainties in the space below in the form $\overline{D}$ +/- ΔD, $\overline{L}$ +/- ΔL and $\overline{M}$ +/- ΔM, where we will use the standard deviations, σ_D, σ_L, and σ_M, as the measure of the uncertainty.

Investigation 1.2 Combining Uncertainties

Investigation 1.2.a Volume

Determine the volume and the uncertainty in the volume, using the method for combining uncertainties in the lab manual. Show your work and record your answer as V +/- ΔV, where V is the volume calculated from the mean values of D and L and ΔV is the uncertainty in the volume measurement as calculated using the method for propagating uncertainties in Section 3.5 of the Introduction to the lab manual.

Investigation 1.2.b Calculating Density

Determine the density and the uncertainty in the density, using the method for combining uncertainties in the lab manual. Show your work and record your answer as ρ +/- $\Delta\rho$, where ρ is the density calculated from the mean values of V and M and $\Delta\rho$ is the uncertainty in the density measurement as calculated using the method for propagating uncertainties in Section 3.5 of the Introduction to the lab manual.

Investigation 1.2.c Interpreting Uncertainty

What material is your cylinder made of? How do you know? Are you sure? How sure? Explain.

Investigation 1.3 Accuracy and Precision

Investigation 1.3.a Fractional and percent uncertainty

Calculate the fractional and percent uncertainty, as in section 3.3 in the Introduction of the lab manual. Show your work and record your results below.

Investigation 1.3.b Accuracy

How accurate is your result? Explain.

Investigation 1.3.c Precision

How precise is your result? Explain.

Could you make your result more precise? How? Explain.

1. Vernier Caliper

It consists of two scales, the main scale (fixed) usually marked in inches on the top and centimeters on the bottom, and the vernier scale which can slide along the fixed main scale. The vernier scale has 10 divisions which are equal to 9 divisions on the main scale, i.e. each division on the vernier scale is 9/10 as long as the smallest main-scale division (see Fig. 1). The difference between the lengths of the smallest division on the two scales is the least amount that can be read accurately and is called the least count of the caliper. Thus, for the caliper shown in Fig. 1, the least count is 1/10 mm or 0.1 mm or 0.01cm.

To measure the length of the specimen, place the specimen between the two jaws. Read the main scale immediately preceding the first mark on the Vernier scale. Then, carefully look for the mark on Vernier scale which coincides with a mark on the main scale. Each division of the Vernier represents 0.1mm and the reading on the Vernier scale is the number of tenths of millimeters that need to be added to the number on the main scale. For example, in Figure 2, the main scale reading is 20 and the fourth mark on the Vernier scale is coinciding with a mark on the main scale. Therefore, we must add the length of 4 divisions of the Vernier scale to the length measured on the main scale. Each division on the Vernier represents the least count of 0.1 mm and thus 4 divisions mean an additional length of 0.4 mm or 0.04 cm. Therefore, the accurate length of the specimen is 20.0mm + 4.0 mm = 24.0mm = 2.04 cm. Note that if one uses a meter stick to measure the length, one can possibly read 20 mm accurately: but anything beyond that must be estimated, i.e. the meter sticks usually do not allow the measurements of the tenths of a millimeter.

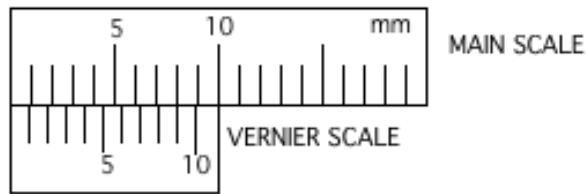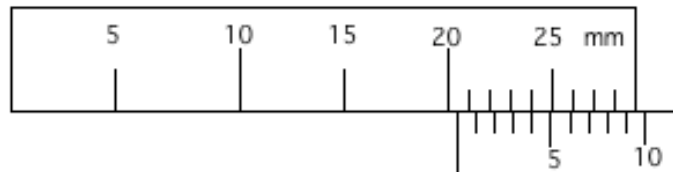What is the length of the specimen in Figure 1?

FIGURE 1

FIGURE 2

It is important to realize that the Vernier caliper you are using may have a correction factor which must be added to or subtracted from each of the measurements. This correction factor can easily be determined if the two zero marks on the main and Vernier scales are not in perfect alignment when the jaws are pressed against each other.

2. Micrometer

The micrometers are designed for accurate measurements (one-hundredth of a millimeter or better) e.g. the diameter of a thin wire or thickness of a thin sheet. As shown in Fig. 3, the specimen is placed between the anvil and spindle and the thimble is moved forward until the specimen is in contact with the anvil. This is best determined when the ratchet R begins to slip. If the thimble is turned through one complete revolution, the distance it moves is known as the pitch of the thimble. The micrometers most commonly used in the laboratory have a pitch of 0.5 mm, i.e. one complete revolution of the thimble corresponds to a linear motion of 0.5 mm. Usually, the sleeve has divisions in millimeters and the thimble has 50 divisions. Thus, each division on the thimble corresponds to 1/50 of 1/2 mm, i.e. 0.01 mm or 0.001 cm, which is the accuracy to which the measurements can be made.

As an example (in Fig. 3) the sleeve shows that the micrometer has been opened 5.5 mm, and since the 28th line on the thimble coincides with the reading line on the sleeve, the accurate measurement is 5.5 + (28 x 0.01) = 5.78 mm.

If the anvil and spindle are closed against each other and the reading on the thimble is not zero, a correction must be added to or subtracted from each measurement. This is an example of the systematic error.
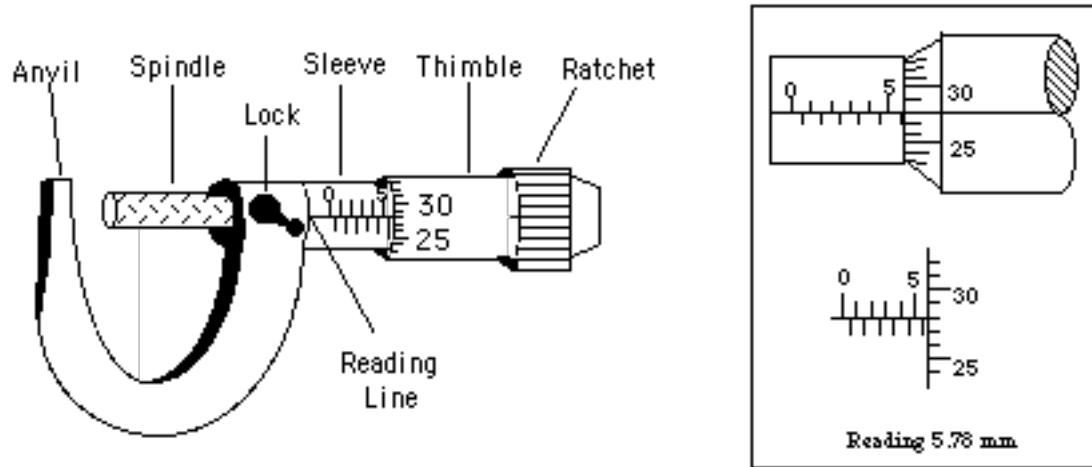

FIGURE 3

Reading 5.78 mm

3.